Shielding of dust grains at the edge of an equilibrium plasma

Huaqiang Luo*
Hyprotech Ltd., 1110 Centre St. N., Calgary, Alberta, Canada T2E 2R2

M. Y. Yu
Institut für Theoretische Physik I, Ruhr-Universität Bochum, D-44780 Bochum, Germany

(Received 5 August 1996; revised manuscript received 17 March 1997)

It is pointed out that in a bounded dusty plasma in local thermodynamic equilibrium, the sheath electric field can very effectively shield *highly* charged ($\geqslant 10^4 e$ for typical low-temperature plasmas) dust grains from the wall region. [S1063-651X(97)01007-6]

PACS number(s): 52.40.Hf, 95.40.+s, 95.55.Pe, 96.50.Gn

Many plasmas in space [1–4] and the laboratory [5–8], as well as charged colloidal fluids [9–12], contain submicron or larger-sized particles that can vary their charge according to the potential difference between their surface and the adjacent medium. They can be massive and highly charged ($10^5 e$ or more), so that new time and space scales appear. Such multicomponent systems are generally referred to as dusty plasmas. In the earlier studies, it is usually assumed that the dusts are cold and stationary, of constant charge, and not in thermodynamic equilibrium. However, for collisional or long-lived systems, the dusts can also be thermalized and achieve Boltzmann density distribution. As dusty plasmas are ubiquitous, they can cover a wide range of parameter values. However, typical space [2,3] and laboratory [5,6] dusty plasmas studied recently have electron temperatures of a few eV, ion and dust (and neutral) temperatures of order 0.1 eV or lower (room temperature), plasma densities of order 10^{10} cm^{-3}, and dust charges up to $10^5 e$. The dust temperature is usually the same as that of the ions or neutrals. For colloidal plasmas [9,10], one usually has room temperature and a relatively small colloidal charge (as low as a few electronic charges).

In this paper, we consider the electrostatic shielding of dust particles near the boundary of a plasma containing Boltzmann electrons, ions, and dust grains. It is shown that *highly* charged grains do not generally affect the plasma sheath electric field [13–15], which is determined mainly by the electrons and ions. However, they form a separate sheathlike structure of their own. The reason for this phenomenon is that, because of their large charge, the dust distribution is very strongly affected by the still-weak electric field at the sheath edge [16]. The dust density sheath (DS) is much narrower than the plasma sheath (PS) supporting it. Thus, the dusts are kept away from the wall region and have a sharp density gradient. Depending on the temperature and behavior of the ions, the classical plasma and/or ion sheath [15] can still exist between the DS and the wall. On the other hand, if the dust charge is low, then the usual Debye-like shielding for a multicomponent plasma occurs. It is also shown that gravity can significantly affect the DS.

We assume that the plasma medium is one-dimensional and in local thermal equilibrium. Since there is usually no or very little direct contact among the dust grains, the latter are assumed to be equilibrated via long-range Coulomb interactions [17] and/or collisions with neutrals. The region of interest lies between $x=0$, where the potential is taken to be zero, and the boundary (wall or the edges of other sheaths) at $x_w>0$. In order to better observe the behavior of the dust grains in the DS, we set the boundary condition at the edge of the unperturbed plasma instead of x_w.

In local thermodynamic equilibrium, the densities of the particles are given by

$$n_j = n_{j0}\exp(-q_j\phi/T_j), \tag{1}$$

where $j=e$, i, and d denote electron, ion, and dust, respectively, and T_j are the corresponding (constant) temperatures. The subscript 0 denotes quantities evaluated at $x\leqslant 0$, where $\phi=0$. Effects arising from the neutral particles shall not be considered.

The average grain charge $q_d(x)$ depends only on the local microscopic electron and ion currents flowing into the grain according to the variation of the potential difference between its surface and the adjacent plasma. Thus, the grain charging process is close to the ion time scale. We invoke the electrostatic probe model [18] for charging, in which the steady-state grain charge is determined by a balance of the grain currents. Accordingly,

$$I_e + I_i = 0, \tag{2}$$

where I_e and I_i are the average microscopic electron and ion currents entering the grain. They are given by

$$I_j = \pi r^2 q_j (8T_j/\pi m_j)^{1/2} n_j W_j, \tag{3}$$

where m_j $(j=e,i)$ are the masses and $r \ll \lambda_e \equiv (T_e/4\pi e^2 n_{e0})^{1/2}$ is the grain radius. For $q_d<0$, corresponding to a negative grain-surface floating potential relative to the adjacent plasma, we have $W_e=\exp(eq_d/rT_e)$ and $W_i = 1 - eq_d/rT_i$, and for $q_d>0$, we have $W_e = 1 + eq_d/rT_e$ and $W_i=\exp(-eq_d/rT_i)$.

Thus, the microscopic grain currents depend on the local electron and ion densities, as well as the potential difference between the dust surface (here at the floating potential) and

*Permanent address: Institute of Physics, Academia Sinica, Beijing 100080, People's Republic of China.

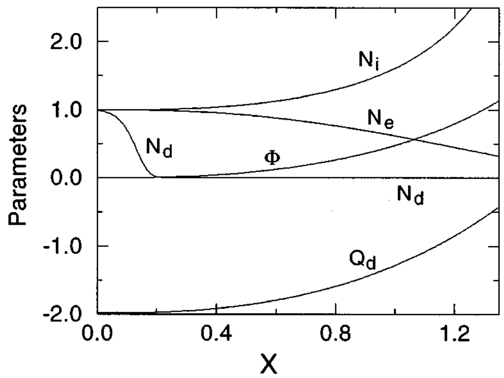

FIG. 1. Dust shielding for $a=b=1$, $f=2$, and $R=500$.

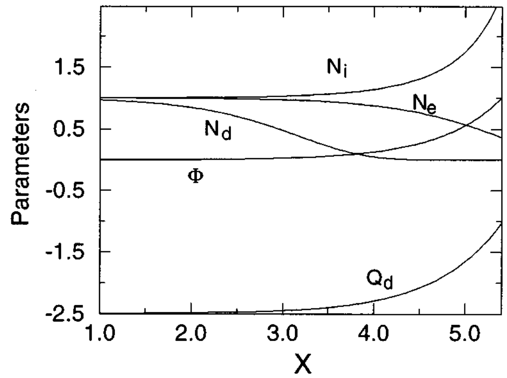

FIG. 2. Dust shielding for $a=b=1$, $f=1.01$, and $R=10$.

the local plasma. A convective charging term ($v_d d_x q_d$, where v_d is the velocity of the dust grain) has been neglected, since the charge variation caused by the grain displacement within the time (on the ion time scale) required for charging the dust to the floating potential is much smaller than that due to the grain currents. According to the probe model, the electrons and ions are continuously being lost to the dust. But they can recombine at the dust surface and the resulting atom then re-enters the plasma and is reionized, such that equilibrium is established. In obtaining Eq. (3), we have assumed that the dusts are sufficiently small and rarefied, so that dust-dust correlations and ion-trajectory interception by neighboring dusts can be ignored. On the other hand, there exists recent theoretical and experimental evidence [8,17] that the Vlasov and fluid descriptions may also be valid for highly correlated grains.

The system is completed by the Poisson equation

$$d_x^2\phi = -4\pi(en_i - en_e + n_d q_d) \tag{4}$$

for $x>0$, and the neutrality condition $en_{i0} - en_{e0} + n_{d0}q_{d0} = 0$ for $x \leqslant 0$.

Equations (2) and (4) are coupled, nonlinear, ordinary differential equations. We introduce the normalized quantities $X = x/\lambda_e$, $\Phi = -e\phi/T_e$, $Q_d = eq_d/rT_e$, and define $f = n_{i0}/n_{e0}$ and $a = T_i/T_e$. Note that the normalization for x is chosen for mathematical convenience. The actual characteristic spatial scale is closer to the (local) dust Debye length, which can be much smaller since q_d is usually large and T_d low. Combining Eqs. (1)–(4), one obtains

$$d_X^2\Phi = -\mathrm{e}^{-\Phi} + f\mathrm{e}^{\Phi/a} - (f-1)\frac{Q_d}{Q_{d0}}\exp(Q_d R\Phi), \tag{5}$$

$$\Phi(Q_d) = \frac{a}{1+a}[-\alpha + Q_d - \ln(a - Q_d)] \tag{6}$$

for $Q_d<0$, where $\alpha = \ln(f\sqrt{m_e/am_i})$ and $R = rT_e^2/e^2T_d$. Within the probe model for dust charging, a solution for *positively* charged grains does not exist [19].

For plasmas with $T_e \sim 1$ eV and $r \sim 1$ μm, one has $100 < R < 1000$ and $Q_d = q_d T_e/eRT_d \sim 10$ for $q_d \sim 10^{2-3}e$ and $T_d = 0.1$ eV. Thus, the last term in Eq. (5), corresponding to the contribution of the dusts, is usually negligible, so that the PS potential is determined solely by the equilibrium electrons and ions. However, because of the large dust charge, the dust density is strongly affected by this potential. In Fig. 1, we present the potential and densities for $a=1$, $f=2$, and $R=500$. We see that in the DS, which is localized at the edge of the PS, the dust density drops sharply, together with a small decrease of the charge magnitude. The dust charge remains constant in the PS, but (the few remaining grains) can become heavily charged toward the wall. Thus, small dust particles are well confined to the center region of the plasma. This phenomenon is consistent with the traditional concept of Debye shielding, as the effective Debye length given by $\lambda = (\lambda_e^{-1} + \lambda_i^{-1} + \lambda_d^{-1})^{-1}$, where λ_i and λ_d are the ion and dust Debye lengths, is dominated by λ_d ($\ll \lambda_{e,i}$). Immediately to the right of the DS, we have $n_i \approx n_e$, $n_d \approx 0$, and $\phi \leqslant 0$, the effective Debye length is given in terms of the ions and electrons, and if the ions are also cooled, a second sheath region, namely, the normal ion sheath [14], can exist between the dust sheath and the wall.

On the other hand, in some space [1] and colloidal [9,10] plasmas, the dust grains can be very large but the charge low, so that R can be much smaller. In this case, the contribution of the dusts to the sheath edge could be significant in the region of interest, where $\Phi \ll 1$. Figure 2 shows the potential

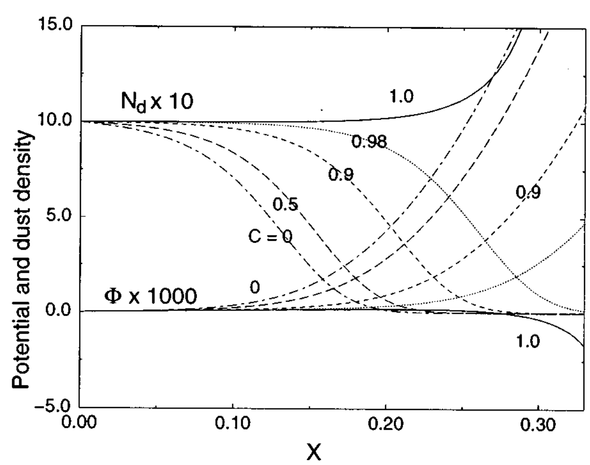

FIG. 3. Dust shielding layer when the gravitational force is included. Only the electrostatic potential and the dust density in the dust shielding region are shown. The other quantities do not vary significantly in this region. For $a=b=1$, $f=2$, $R=500$.

and density distributions for $a=1$, $f=1.01$, and $R=10$. We see here that the sheath electric field is affected by all the particles and the DS is no longer clearly separable. We also note that the DS is considerably wider than that in Fig. 1. Here, the sheath profile (except for the variation of Q_d) is similar to that of a two-ion plasma.

If the surface exerts a gravitational force on the dust grains, the result can be significantly altered. Adding a gravitational potential term to n_d results in the appearance of a term $+CX$, where $C=\lambda_e m_d g/T_d$ and g is the gravitational acceleration (here taken to be the Earth's), to the exponential in the last term of Eq. (5). For heavy particles with $m_d \approx 10^{-9}$ g ($C \gg 1$), the gravity dominates over the electric forces in determining the dust density and the electric potential. For lighter particles with $m_d \approx 10^{-15}$ g, the effect of gravity can be of the same order as or smaller than the electric force. In Fig. 3, we show a case where gravitational effects are included. Here, only the DS (small X) region is shown. It is clear that for the given parameters, confinement by the PS becomes impossible when C is close to unity or larger.

Recently, there is much interest in the theory of dust crystals [20,21], which have also been observed in numerical simulations [22,23] and laboratory experiments [24,25]. Most existing theories and simulations on crystal formation (the phase transition problem) assume that near the transition point the system should be in or near thermal equilibrium. Thus, electrostatic dust confinement of an equilibrium dusty plasma discussed here is particularly relevant. However, for investigating crystal formation, other effects, such as charging asymmetry arising from ion-orbit intersection by neighboring dust grains, must be taken into consideration. On the other hand, the result that gravitational effects can be very important for dusts in plasmas is especially significant for experiments on dust crystal formation in weightless conditions [26].

Finally, we point out that the result here is of significance also for charged colloidal systems [11,12]. In particular, it is demonstrated that highly charged colloids can be well shielded from the container wall by the DS electric field, so that their physical and chemical interactions with the wall could be ignored.

This work has been partially supported by the Association for Plasma Studies of China and the Sonderforschungsbereich 191 Niedertemperaturplasmen. One of the authors (H.L.) would like to thank the K. C. Wong Education Foundation, Hong Kong, for financial support.

---

[1] L. Spitzer, *Diffuse Matter in Space* (Interscience, New York, 1968), Chap. 4.
[2] E. C. Whipple, Rep. Prog. Phys. **44**, 1198 (1981).
[3] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).
[4] P. A. Bernhardt, G. Ganguli, M. C. Kelley, and W. E. Swartz, J. Geophys. Res. **100**, 23 811 (1995).
[5] D. B. Graves, J. E. Daugherty, M. D. Kilgore, and R. K. Porteous, Plasma Sources Sci. Technol. **3**, 433 (1994).
[6] A. Melzer, T. Trottenberg, and A. Piel, Phys. Lett. A **191**, 301 (1994).
[7] S. Robertson, Phys. Plasmas **2**, 2200 (1995), and references therein.
[8] A. Barkan, R. L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[9] D. W. Schaefer and B. J. Ackerson, Phys. Rev. Lett. **25**, 1448 (1975); N. A. Clark, A. J. Hurd, and B. J. Ackerson, Nature (London) **281**, 57 (1979).
[10] K. Kremer, M. O. Robbins, and G. S. Grest, Phys. Rev. Lett. **57**, 2694 (1986); E. B. Sirota, H. D. Ou-Yang, S. K. Sinka, and P. M. Chaika, Phys. Rev. Lett. **62**, 1524 (1989).
[11] D. Henderson, in *Trends in Interfacial Electrochemistry*, edited by A. F. Silva (Reidel, New York, 1986), p. 473.
[12] S. L. Carne and G. M. Torrie, in *Advances in Chemical Physics LVI*, edited by I. Prigogine and S. A. Rice (Wiley, New York, 1984), p. 142.
[13] P. C. Clemmow and J. P. Dougherty, *Electrodynamics of Particles and Plasmas* (Addison-Wesley, Reading, MA, 1969), p. 236.
[14] F. F. Chen, *Introduction to Plasma Physics* (Plenum, New York, 1974), Chaps. 1 and 8.
[15] Here, plasma sheath refers to Debye-like shielding by Boltzmann electrons and ions, and ion sheath refers to shielding by Boltzmann electrons and cold ions.
[16] Here, ''edge'' refers to the region immediately adjacent to the unperturbed plasma.
[17] X. Wang and A. Bhattacharjee, Phys. Plasmas **3**, 1189 (1996).
[18] F. F. Chen, in *Plasma Diagnostic Techniques*, edited by R. H. Huddlestone and S. L. Leonard (Academic, New York, 1965), Chap. 4.
[19] With the probe model, one would obtain

$$\Phi(Q_d)=\frac{a}{1+a}\left\{-\alpha-\frac{Q_d}{a}-\ln[-(1+Q_d)]\right\} \tag{7}$$

for $Q_d>0$. It is obvious from this equation that a well-behaved solution for *positively* charged grains cannot exist. In fact, even the complete-equilibrium state ($\Phi=0$, as should be the case in the region $x<0$) does not exist for positive grains. This is because for such an equilibrium (ions also satisfying the Boltzmann density distribution) there would be more electrons in the neighborhood of a grain. Since no secondary electron-emission mechanism is included in the model, more electrons (than ions) will then hit the grain, until Q_d becomes negative.
[20] H. Ikezi, Phys. Fluids **29**, 1764 (1986).
[21] S. V. Vladimirov and M. Nambu, Phys. Rev. E **52**, 2172 (1995).
[22] R. T. Farouki and S. Hamaguchi, Appl. Phys. Lett. **61**, 2973 (1992).
[23] X. H. Zheng and J. C. Earnshaw, Phys. Rev. Lett. **75**, 4214 (1995).
[24] J. H. Chu and Lin I, Phys. Rev. Lett. **72**, 4009 (1994).
[25] H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher, and D. Möhlmann, Phys. Rev. Lett. **73**, 652 (1994).
[26] H. Thomas, G. E. Morfill, H. Rothermel, and M. Zuzik (unpublished).